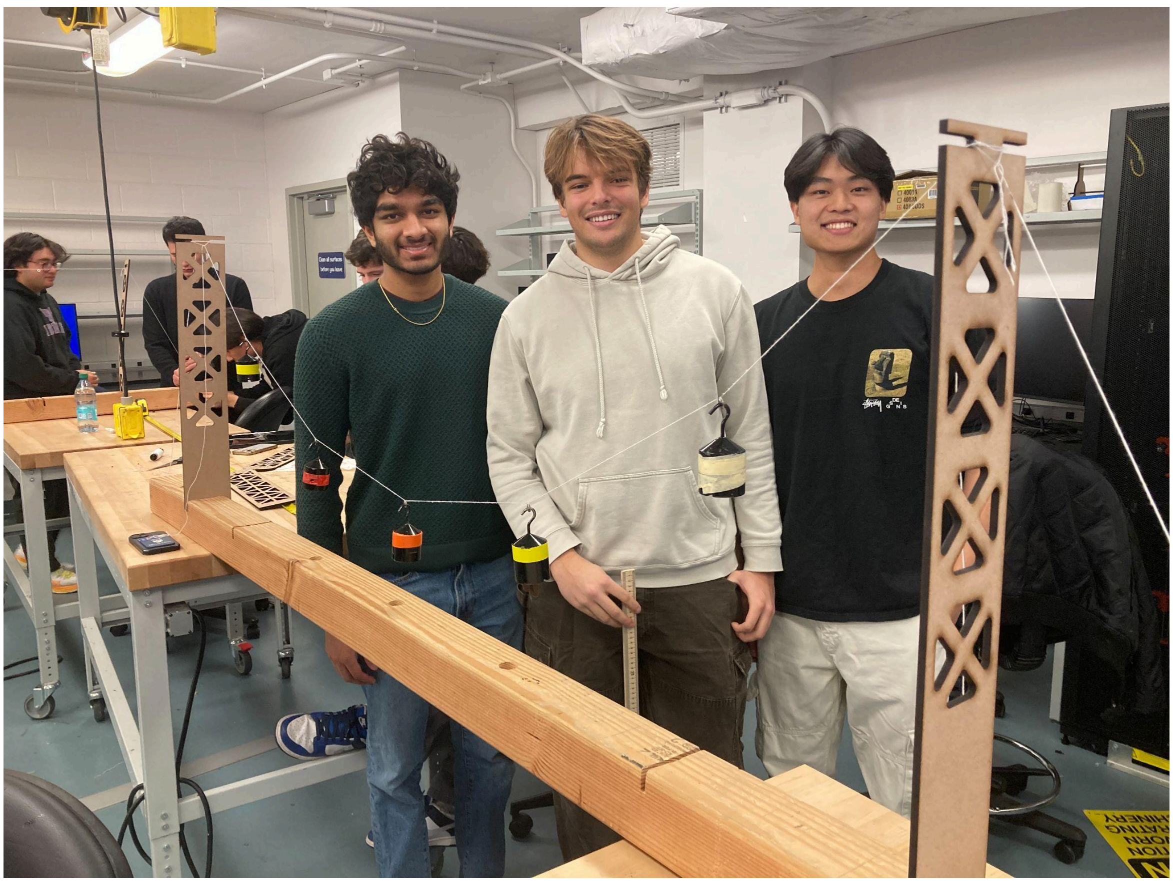
Suspension Bridge Project



Adam Laycock, Sridhar Shenoy, Alex Zhao

Introduction and Requirements

In this project, we were tasked with designing and building a model suspension bridge that could hold a given amount of weight hung from the suspension cables. The following criteria were required:

- The entire bridge spanned 8 feet, with an option of two configurations: the towers being either 4 or 6 feet apart
- The two towers were to be laser cut from either ¼" or ⅛" MDF, and were to be no larger than 4 inches wide and 24 inches tall
- The inner cable was constructed of kite twine, and could be made of multiple strands
- The inner cable was not allowed to sag below a 6 inch clearance from the ground
- The bridge would carry a load of either 5 (wide span) or 7 (narrow span) 0.5 kg weights spread evenly across the inner cables
- The outer cables would also be made of kite twine

The goal was to design a bridge that fit these requirements while minimizing cost in MEAM Cost Units (MCUs). The cost was calculated using the following criteria:

- By default, the narrow and wide span configurations cost 40 and 10 MCUs, respectively
- The marginal cost of each inner cable strand increased as a fibonacci sequence (10, 20, 30, 50…)
- The cost of the tower was decided using the formula: (above-base area of a single tower face in square centimeters)*(thickness cost factor) where the thickness cost factor was 0.5 for ¼" MDF and 0.2 for ⅛" MDF
- Voted by peers, the most aesthetically pleasing bridge would receive a credit of 20 MCUs and the runner up would receive 10
- There was no cost for additional outer cables

Overall Design Strategy

The first question we had to address was whether to attempt the wide or narrow span configuration. We did not have much direction, but we hypothesized that we could make the wide span configuration cheaper given the default costs, as we did not think the narrow span would make up for the cost discrepancy elsewhere. We also decided that we would aim for structural integrity rather than aesthetics, since the reward for aesthetics seemed low given the difficulty and risk of attempting to build the prettiest bridge. Additionally, we elected to use ¼" MDF for the towers to keep the tower safe from buckling and fracture, and used 2 intertwined strands for the outer cable because they came at zero cost.

After some brief thought and discussion, we decided that one of the key aspects of our design would be maximizing sag in the cable. We knew that this was one of the most likely failure modes, since the cable sagging below the 6 inch clearance would automatically result in failure. However, maximizing the sag, and therefore maximizing the angle of inclination at the inner cable's attachment points was crucial to minimizing the stress on every component of the bridge.

Since the vertical component of tension at the top of the inner cables would be the same (half the total load due to symmetry) regardless of the angle, the horizontal component would increase as the angle decreased [Figure 1].

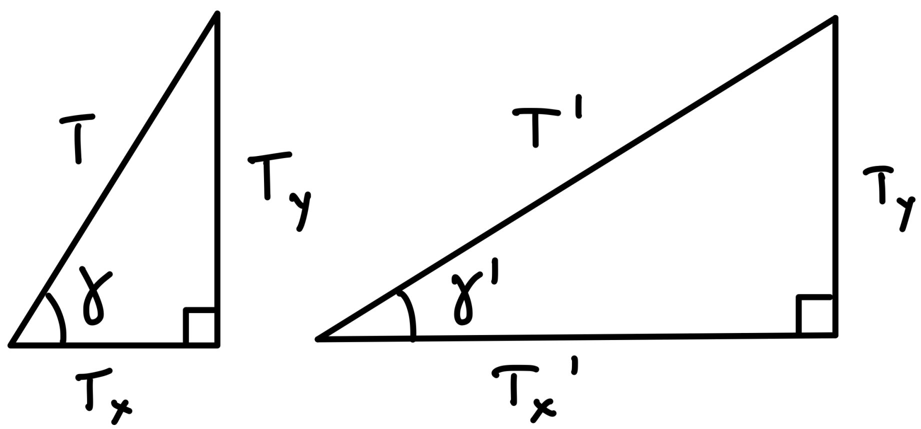


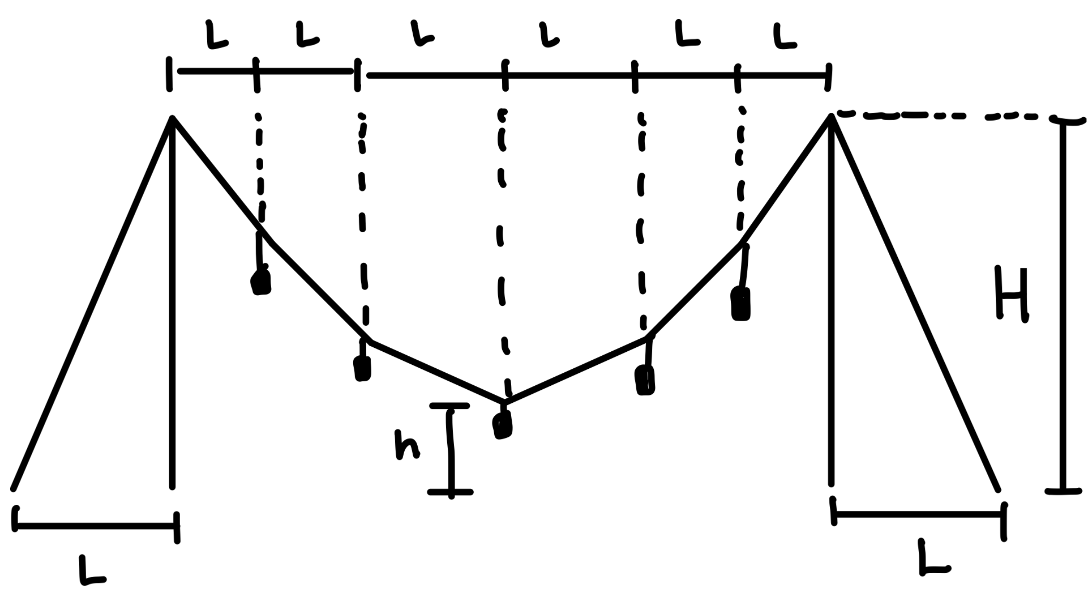
Figure 1: Drastic increase in tension as gamma (angle of inclination) decreases

In order to keep the towers vertical, the outer cables would also require more tension for a smaller angle of inclination, therefore increasing the total force the towers were subjected to as well. This "domino effect" made it clear to us that maximizing cable sag would be essential for safety and minimizing total cost.

Final Design Summary

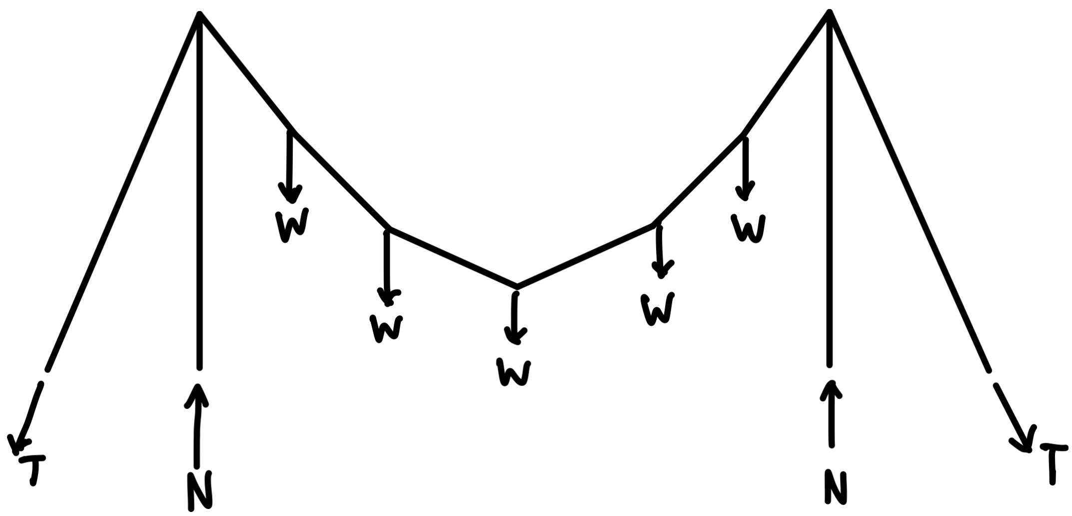
Keeping in mind safety and cost minimization, this was our final design (not to scale):

Figure 2: Drawing of bridge with dimensions



H = 23 in. h = 8 in. L = 12 in. Total Cable Length = 79.15 in.

Figure 3: Free body diagram of entire bridge (neglecting weight of towers and cables)



N = 39.44 N W = 4.91 N T= 30.64 N

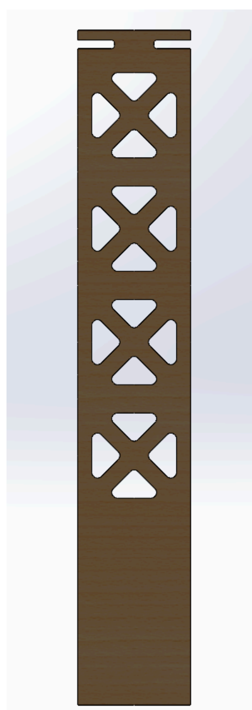


Figure 4: Final design of laser cut tower

Subsystem Design Process

Tower: Alex Zhao

As the Tower Officer, my responsibility was to design the tower and ensure its structural integrity given all of the loads that the Cable Officer would calculate. I was also the primary person in charge of reducing the cost as much as possible for our team as the surface area is the most variable component for the cost.

I started the design process with the key parameters in mind, such as maintaining a width that would fit inside the slot and setting my height with the Cable Officer in order to ensure that key parameters were defined and constrained. Once I established the key constraints for our tower, I began to look at structural analysis and also determining the shape and cutouts to make. It is important to note that I approached this from a top down method, where I had essentially a flat plank as my tower in the beginning. I then began exploring pocket geometries and seeing how I could reduce the surface area as much as possible. I designed the pockets to have no sharp corners in order to reduce stress concentrations, and I also ensured that the design was axis symmetric about the y-axis in order to ensure that the load would be evenly distributed. Ultimately I decided on a sort of thick truss design where the pockets formed these “X” shapes to allow for adequate support and to prevent buckling [Figure 4].

Inner Cable: Adam Laycock

As Cable Officer, my responsibility was to calculate the tensions, angles, and elongations in each of the cable segments. Keeping in mind that one of our main goals was to maximize cable sag, I calculated these values for an 8 inch clearance. This way, even if the cable elongated more than expected, it would still be above the 6 inch clearance.

I approximated the shape of the cable as a parabola and solved for the derivative at the point of connection. In hindsight, a catenary curve would have been more ideal, but the parabola was extremely similar in the relevant domain. Once I had the derivative, I was able to find the approximate angle of inclination of the top segment [Figure 5]. Along with the angle, knowing that the vertical component of tension was equal to half the total supported weight allowed me to solve for the tension in the cable.

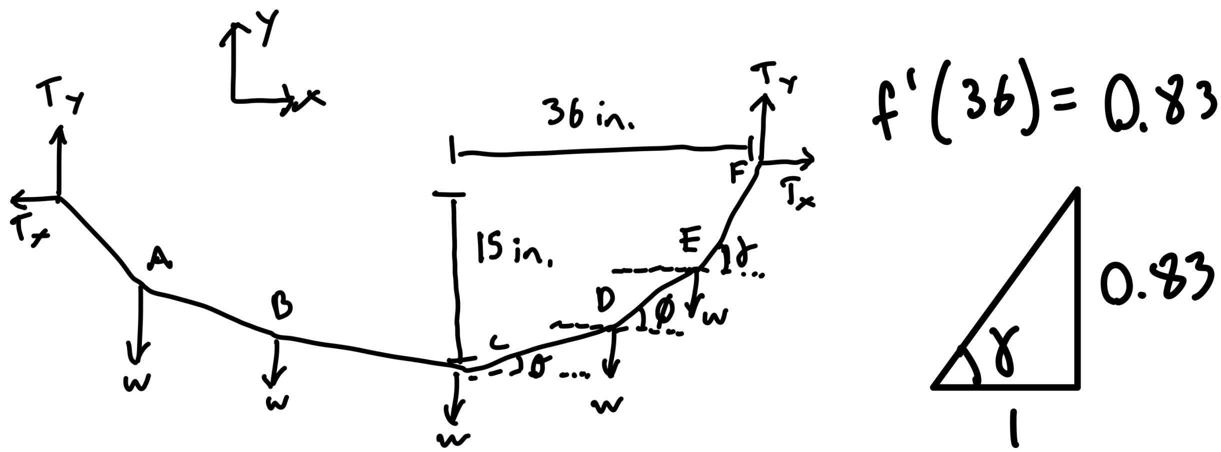


Figure 5: FBD of inner cable, the derivative of the parabolic function, and its relation to gamma (angle of inclination).

After solving for the tension in segment EF, I drew a free body diagram of point C. Summing the forces in the y-direction to zero, I was able to solve for the vertical component of tension in CD. The x-component was less straightforward; I noticed that if I made a section cut at point C, the only horizontal forces acting on the cable were T_x and the x-component of tension in BC. In fact, making a section cut at any two points on the cable gave the same fact: the x-components of tension were the same at any point in the cable. Using this, I was able to solve for T_{CD} and T_{DE} as well as all of the unknown angles. Once I had the tensions, I used the MTS data [Figure 11] to find each segment's corresponding elongation. All of my results were recorded in the table below [Figure 6].

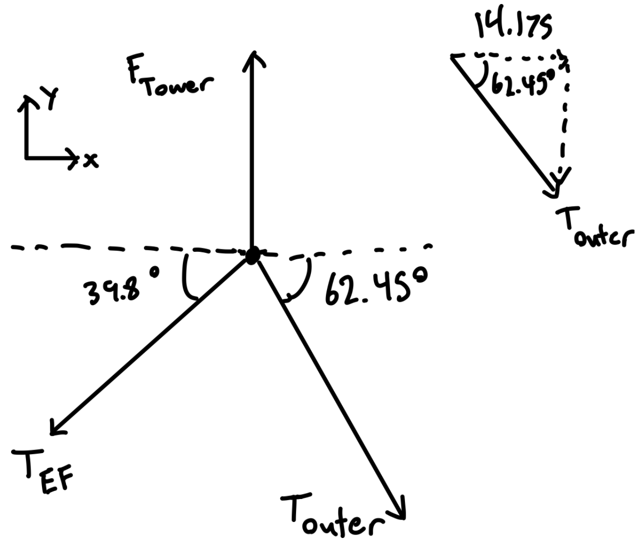
Segment	Angle	Tension	% Elongation	Length (Stretched)	Length (Original)
CD	$\theta = 9.34^{\circ}$	14.91 N	3.6	12.16 in.	11.74 in.
DE	$\phi = 26.6^{\circ}$	16.45 N	4.0	13.42 in.	12.90 in.
EF	$\gamma = 39.8^{\circ}$	19.16 N	4.6	15.62 in.	15.62 in.

Figure 6: Table displaying tensions, angles, and elongations of all inner cable segments

Outer Cables: Sridhar Shenoy

My tasks as project manager were to calculate the tensions in the outer cables, and design either a 3D printed part or a shape to be laser cut for the cables to be tied around.

To calculate the tension in each of the outer cables, I drew a free body diagram of the attachment point using Adam's calculation of the tension in the top segment of the inner cable [Figure 8]. In order to keep the top of the tower from bending one way or the other, the x-component of tension in each outer cable had to equal the x-component of tension at the top of the inner cable. I knew the angle between the tower and the outer cable due to the positions of the attachment points, and I was able to use it to solve for the tension in the outer cables (30.64 N).



Figures 7 & 8: Free body diagram and laser cut shape for cable attachment

As for the attachment point, I decided to laser cut a shape into the tower. A 3D printed part seemed like it would be unsturdy and unnecessary. I designed a pretty simple “neck-like” shape of which the cables could be wrapped around [Figure 9].

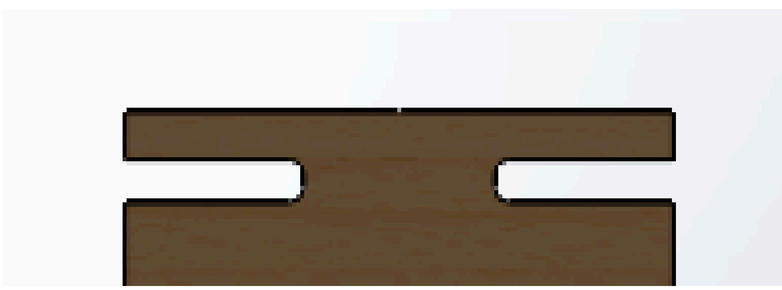


Figure 9: Close-up of mounting cutout for cable attachment

Subsystem Integration

After careful analysis of each subsystem, we had to integrate all of our findings and designs into our final design to be presented on test day. The main concerns we needed to address were:

- How to accurately tension the outer cables
- How many strands of twine to use
- What kind of knots to use

It was required that on test day, the outer cables must be tensioned before attaching the inner cable. Without any instruments to measure the tension in the cable, this was difficult. Our solution was to use the MTS data [Figure 11] to find the elongation at which the cable was at the proper tension. During testing, we found that the twine could withstand a maximum load of 54 N before breaking. The maximum tension in our inner cable was just under 20 N. This was enough of a safety factor for us to feel comfortable using one strand of twine. However, since the outer cables came at no extra cost, we decided to use two strands for each outer cable just to be extra safe. We used square knots [Figure 10] to tie the cables, as they are easy to learn and stronger than granny knots (how most people tie a double knot).

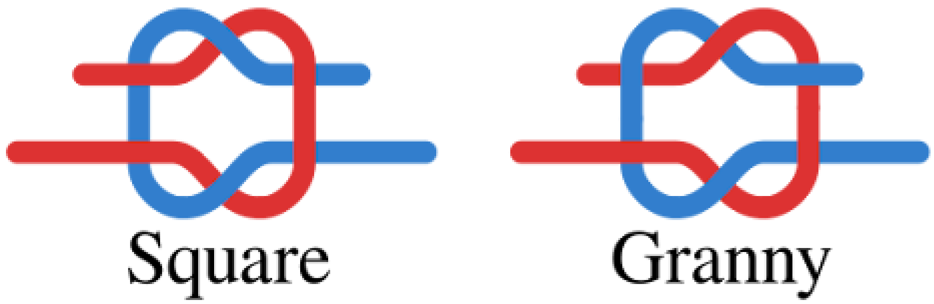


Figure 10: Square vs. Granny Knot

Experiments and Testing

Twine Tensile Test

During the lab section, we used the MTS machine to obtain values for the elongation of the twine at different tensile loads. We fastened two allen wrenches to the machine and tied a string around both wrenches. We then increased the force until the single strand of twine snapped.

Once we had the raw data, we plotted it in excel to determine a relationship between load and elongation. The data revealed that this relationship was linear, and the twine reached an ultimate tensile load of 54 N with about a 16.7% elongation. The data was not perfect though, as the knots kept slipping throughout the test. Below is a graph of load vs. percent elongation [Figure 11], in which the slippage can be observed as sharp drops in load. Regardless, this data proved to be extremely useful in our design process.

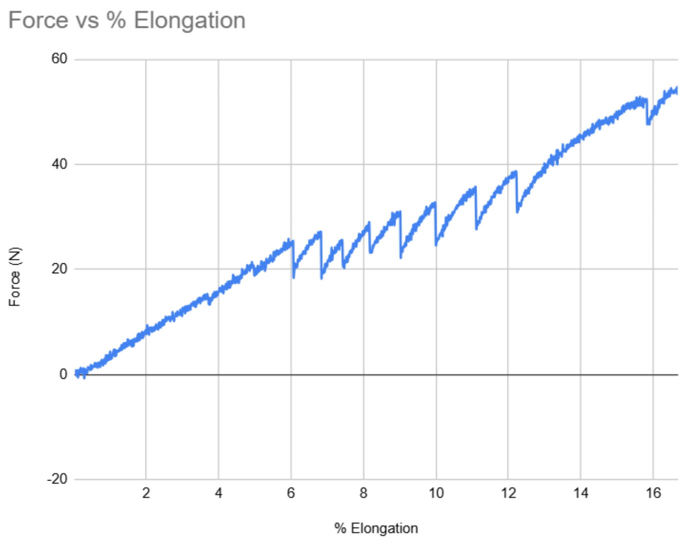


Figure 11: Force vs. Percent Elongation graph for twine tensile test

Outer Cable Tensioning

It was required that the outer cable had to be tensioned prior to hanging any weights. This was one of the biggest challenges of this project, but it was crucial to a successful demonstration. From our MTS data, we knew that at our desired tension (about 30 N) would have a given elongation of about 3.6 percent, given that there we used two strands of twine. So, prior to test day, we taped our pieces of twine to a table, made a mark at the original length, and stretched them. Once the twine was stretched so that the original mark had been displaced by 0.93 inches (3.6% the original length), we made another mark at the original length [Figure 12]. On test day, this served as a guide for ensuring we had the proper amount of tension in the outer cables.

Figure 12: Mark at original length and at a 3.6% displacement

Finite Element Analysis Validation

The Tower Officer ran Finite Element Analysis (FEA) on the forces found from the Cable Officer and FEA was run to ensure that the tower would not buckle. There were a couple of working assumptions while approaching this validation, with the first being the use of a similar material profile in order to simulate that of MDF. We chose Balsa wood as a substitute material since the yield and compressive strength seemed close to the material properties of MDF that was found online.

To run the buckling analysis, I assumed that the bottom lip of the MDF sheet was fixed, while there was a perfectly vertical force being applied downwards at the top of the tower. Clearly there were a lot of simplifying assumptions made, however in an ideal physical world, this simulation would represent the forces that are experienced by the tower. I chose not to test lateral loads as the bending properties of MDF and balsa were more dissimilar, and I did not want to introduce another layer of factor of safety which would likely make our tower too conservative.

From the forces that the other members of the team calculated, a vertical downward load of 39.44 N was applied to the top of this tower where the string interacts with it, we obtained a factor of safety of 1.48, as we were trying to aim for a factor of safety of at least 1.3, which we successfully accomplished. These results can be seen below in Figures 13 and 14.

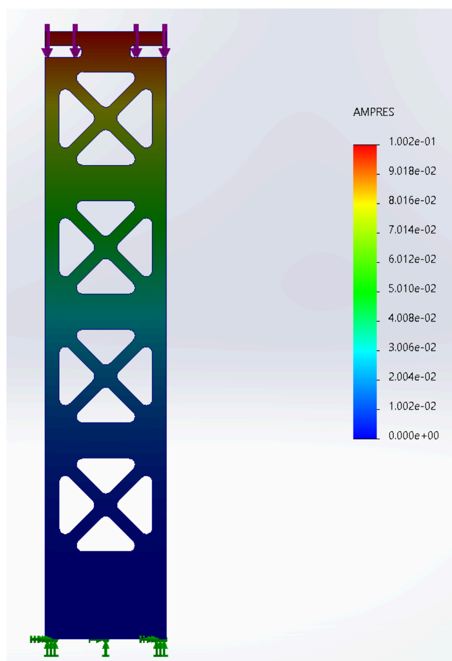


Figure 13: FEA of buckling analysis on the tower

Mode No.	Buckling Factor of Safety
1	1.4853

Figure 14: Obtained factor of safety from the FEA

Test Day: Results and Analysis
On test day, our bridge successfully fulfilled all of the requirements. The cable sagged all the way down to a 6.5 inch clearance, narrowly satisfying the required 6 inch clearance. While we technically achieved our goal of maximizing sag, therefore minimizing forces on the bridge, this result was a bit close for comfort, and it was alarming that our clearance was 1.5 inches less than what we predicted. On the other hand, our method of tensioning the outer cables seemed to work as expected, as we observed very minimal bending at the top of the tower. This indicated that the horizontal components of tension in the inner and outer cables were quite close in magnitude.

During the demonstration, we observed no sign of buckling at all in the towers. If we based our understanding of the situation purely off the FEA simulation, then this would make sense, but it is also important to note that once laser cut, it was obvious that the towers were much stronger under applied loads than they appeared to be within the simulations.

Figure 15: Successful demonstration on test day

Through watching other teams' demonstrations, we gained some valuable insights into the core principles of this project. For example, a common downfall was a lack of tension in the outer cables, resulting in bending of the towers and unpredicted cable sag. But perhaps the most interesting and insightful observation we made had to do with the effects of the inner cable length. Many groups had an inner cable that was much shorter than ours, and their cables were taut even before adding any of the weights. Initially, we thought this approach was understandable, as failing to maintain a 6 inch clearance would result in an automatic failure. However, we noticed a common theme with shorter inner cables: not only did the cables seem to have more tension (sometimes resulting in failure), but they also seemed to have a much greater vertical displacement than ours. This was likely due to the fact that the increased tension elongated the cables more, so ultimately this design choice provided a limited benefit while significantly reducing the factor of safety of the entire bridge.

Sources of Error and Possible Improvements

Our design was ultimately successful, but the clearance with the final load made the margin for failure incredibly small. We measured a 6.5 in clearance with the lowest point of the cable, but this was a lot closer to the 6 in cutoff than we had expected with our calculations. This error could be for a number of reasons, the most plausible being the method of cable attachment to the towers. Although the twine was marked to ensure a precise attachment, actually tying the twine so it resulted in an accurate length proved to be quite difficult. Because the loop was not tightly wrapped around the tower [Figure 16], we had to estimate our desired length. This likely added length to our cable, decreasing the clearance. A possible solution to this that we observed in another team's design is a hole which the cable could pass through with a knot tied on the opposite side.

Figure 16: Twine loosely looped around laser cut attachment point

Another source of error could be the angle approximation used in the inner cable subsystem design. A parabola was used, but in reality, the loaded cable would be best represented by a piecewise function. However, this would be at best extremely difficult to approximate, so a more conservative angle estimate would probably be a more feasible solution.

The buckling analysis performed through the FEA simulation also seemed to be inaccurate compared to the actual results, but this can be explained when we realize that the material profile we input and the one we actually tested were fundamentally different. The balsa wood profile was likely far weaker than that of the specific MDF that was available in the RPL, and thus our design was far too conservative, especially when looking at other teams. Through the analysis on FEA, if we had increased the pockets just by 10%, the factor of safety would immediately plummet and thus according to the simulation, we had to maintain less pockets in order to be safe. In reality, we could probably reduce the material usage in our towers by quite a large factor and still conduct a successful demonstration. This would be monumental in reducing cost, since the area of the towers was the largest contributor to our final cost. If we redesigned our towers, we would probably conduct compressive strength tests using the MTS machine instead of FEA analysis, as this would be more applicable to our material and real world conditions.

Conclusion

By participating in this project, we were able to delve into the design process and gain valuable insights into the application of concepts learned in MEAM 2100. Through calculations, testing, and observations of other groups, we obtained some notable takeaways. For example, maximizing cable sag in a suspension bridge results in a reduction of forces experienced by all components of the bridge. Not only that, but we found that attempting to achieve a safe clearance buffer by minimizing cable sag is not very effective, as the increased tension results in more elongation. This project also reinforced the significance of laboratory testing, as our MTS data was extremely useful, and an additional MTS test of the tower would have led us to massively reduce cost. Ultimately, this project served as a comprehensive introduction to the engineering design process and we will continue to use what we have learned moving forward.